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# MODELING OF CRITICAL PHENOMENA IN PLEBANSKI-DEMIANSKI ISLAND SYSTEMS TAKING INTO ACCOUNT ROTATION AND ACCELERATION

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Mathematical modeling of critical phenomena in systems described by island models is relevant. This is due to the fact that island models can be used to predict unstable states of elementary particles, which expands the possibilities of predicting processes in modern nuclear power engineering. Modeling of the vacuum-matter phase transition on the analysis of critical phenomena in island systems can be performed. The main provisions of the theory of catastrophes of Thom and phase transitions of Landau were used to predict the possibility of occurrence of critical phenomena in island systems. Calculations of phase states were made using the differential-topological approach. The results of the calculations indicate the possibility of fulfilling the conditions of phase transitions of the second kind, when two different phases of the system can coexist simultaneously. The system becomes unstable and can pass from one stable phase to another even with small fluctuations under such conditions. To calculate the phase states of island systems, the Plebanski-Demianski island model was studied taking into account rotation and acceleration.
**Keywords**: coexistence of phases, critical phenomena, phase spaces, island systems, matrix determinant.

**Introduction.** The prediction of the stability of island systems is directly related to the modeling of the stability of elementary particles [1 – 3], however, the problems of violation of the stability of phase states of such systems have not been sufficiently studied [4 – 7]. There are no studies that examine the conditions for the emergence of coexisting phases. However, the process of formation of critical spaces and spaces of coexistence of phases of different orders is possible in island systems under certain conditions [8 – 12]. Such states of the system can lead to a violation of stability [13 – 17]. The space of coexistence of phases arises when one stable state coexists with another stable state [18]. The appearance of such a space is a phase transition of the first kind, determined by Maxwell's principle. Two (or more) global minima of the potential function in such a space have the same depth [19]. The stable phase can become unstable at some points of the studied space, forming a bifurcation subspace [20]. Two phases in the critical region may become identical at some values of the order parameter. The emergence of identical phases may lead to the formation of a critical space of order two. In the presence of three or four identical phases, critical spaces of order three or four are formed, respectively [21]. The equation of state of the system specifies some n-dimensional manifold when describing phase transitions in the corresponding space. Thom's catastrophe theory can be used to estimate the features of the potential function of a self-organizing system in the case of using one order parameter. With this approach, catastrophe theory is considered as a generalized form of the Ginzburg-Landau phase transition theory [18]. However, to assess the conditions for the emergence of coexisting phases and to describe possible phase transitions in multicomponent systems, it is necessary to use approaches that allow one to analyze the features of potential functions of several order parameters. The properties of the Plebanski-Demianski island model [22] were studied using a non-orthogonal gradient tetrad. The Plebanski-

Demianski model (eight-parameter solution of vacuum Einstein-Maxwell equations) was studied as a model of Schwarzschild, which contains a local singularity [19, 20]. The technique of computer modeling of the process of formation of critical spaces in complex multicomponent systems based on the use of a differential topological approach can be used.

**Lagrangian of the Plebanski-Demianski island model.** In present communication we study the island model with Plebanski-Demianski type metric in the framework of Riemannian geometry with curvature flows of vacuum space-time. The difference between the equations obtained in our approach in the framework of this model and equations obtained in the framework of the Plebanski-Demianski model [22] is discussed for proposed of non-singular model obtaining. The class of space-time with signature (+, +, +, –) in the Plebiansky-Demiansky model is investigated,

described in coordinates $x^i = (p, q, \sigma, \tau)$:

$$ds^2 = \frac{1}{(p+q)^2}\{\frac{1+(pq)^2}{P}dp^2 + \frac{P}{1+(pq)^2}(d\sigma + q^2 d\tau)^2 + \frac{1+(pq)^2}{Q}dq^2 - \frac{Q}{1+(pq)^2}(d\tau - p^2 d\sigma)^2\} \quad (1)$$

where $P = P(p)$ and $Q = Q(q)$ are arbitrary structure functions depending on $p$ and $q$, respectively.

The Lagrangian was constructed on the basis of gradient vectors to model critical phenomena in the system under consideration (1):

$$m_i = \frac{\partial x^0}{\partial x^i} = (1,0,0,0), \quad n_i = \frac{\partial x^1}{\partial x^i} = (0,1,0,0),$$
$$p_i = \frac{\partial x^2}{\partial x^i} = (0,0,1,0), \quad s_i = \frac{\partial x^3}{\partial x^i} = (0,0,0,1) \quad (2)$$

The local basis (2) in this case in the general case does not necessarily have to be orthogonal. The proposed approach allows us to obtain a metric tensor in the form of a bilinear combination of basis vectors, the coefficients of which will be functions, and the Lagrangian as a combination of these functions and their first derivatives. Thus, within the framework of the standard field theory, it is possible to obtain a Lagrangian for a rotating charged uniformly accelerated mass in GTR.

From (1) and (2) it follows that

$$x^0 = \tau, \quad x^1 = \sigma, \quad x^2 = p, \quad x^3 = q, \quad (3)$$

and the signature of the space has the form (–, –, –,+). Then, taking into account (3), expression (1) takes the form

$$ds^2 = \frac{1}{(p+q)^2}\{\frac{Q}{1+(pq)^2}(dx^0 - p^2 dx^1)^2 - \frac{P}{1+(pq)^2}(dx^1 + q^2 dx^0)^2$$
$$- \frac{1+(pq)^2}{P}dx^{2^2} - \frac{1+(pq)^2}{Q}dx^{3^2}\} \quad (4)$$

The metric tensor corresponding to (1) will then have the form [23]:

$$g_{ik} = A m_i m_k - B(m_i n_k + m_k n_i) - C n_i n_k - D p_i p_k - F s_i s_k,$$

where

$$A = \frac{Q - q^2 P}{(p+q)^2(1+(pq)^2)}, \quad B = 2\frac{p^2 Q + q^2 P}{(p+q)^2(1+(pq)^2)}, \quad (5)$$

$$C = -\frac{p^4 Q - P}{(p+q)^2(1+(pq)^2)}, \quad D = \frac{1+(pq)^2}{(p+q)^2 P}, \quad F = \frac{1+(pq)^2}{(p+q)^2 Q} \quad (6)$$

are functions of coordinates. The components of the metric tensor and basis vectors with superscripts were found in the form:

$$g_{kl}=\begin{pmatrix} A & -B & 0 & 0 \\ -B & -C & 0 & 0 \\ 0 & 0 & -D & 0 \\ 0 & 0 & 0 & -F \end{pmatrix},\quad g^{ik}=\begin{pmatrix} \frac{AC}{A(B^2+AC)} & -\frac{B}{AC+B^2} & 0 & 0 \\ -\frac{B}{B^2+AC} & -\frac{A}{AC+B^2} & 0 & 0 \\ 0 & 0 & -1/D & 0 \\ 0 & 0 & 0 & -1/F \end{pmatrix} \tag{7}$$

$$m^i=\left(\frac{AC}{A(B^2+AC)};-\frac{B}{B^2+AC};0;0\right),\ n^i=\left(-\frac{B}{AC+B^2};-\frac{A}{AC+B^2};0;0\right),$$
$$p^i=\left(0;0;-\frac{1}{D};0\right),\qquad s^i=\left(0;0;0;-\frac{1}{F}\right) \tag{8}$$

The Lagrangian of system (1) was represented as:

$$L=G=\Gamma^m_{il}\Gamma^l_{km}g^{ik}-\Gamma^m_{nm}\Gamma^n_{il}g^{il}=\Gamma^m_{il}\overset{(s)}{\Pi}{}^{.il}_{m}-\overset{(as)}{\Pi}{}^{.m}_{nm}\Gamma^n_{il}g^{il}, \tag{9}$$

where

$$\Gamma^m_{il}\overset{(s)}{\Pi}{}^{.il}_{m}=\frac{1}{4}\{-\frac{2(D_mp^m)^2}{D}-\frac{2(F_ms^m)^2}{F}-\frac{1}{(B^2+AC)^2}[A^mA_mC^2+4A^mB_m-2A^mC_mB^2+2B^mB_m(B^2-AC)+4B^mC_mAB+C_mC^mA^2]-\frac{D_mD^m}{D^2}-\frac{F_mF^m}{F^2}\};$$

$$\Gamma^m_{il}\overset{(s)}{\Pi}{}^{.il}_{m}=\frac{1}{4}\{\frac{F^{\Delta^2}}{F^3}+\frac{D^{*2}}{D^3}+\frac{D^{\Delta^2}}{FD^2}+\frac{F^{*2}}{DF^2}-\frac{2}{B^2+AC}(\frac{B^{*2}}{D}+\frac{B^{\Delta^2}}{F})+\frac{1}{(B^2+AC)^2}[A^2(\frac{C^{*2}}{D}+\frac{C^{\Delta^2}}{F})+C^2(\frac{A^{*2}}{D}+\frac{A^{\Delta^2}}{F})-2B^2(\frac{A^*C^*}{D}+\frac{A^{\Delta}C^{\Delta}}{F})+4BC(\frac{A^*B^*}{D}+\frac{A^{\Delta}B^{\Delta}}{F})+4AB(\frac{B^*C^*}{D}+\frac{B^{\Delta}C^{\Delta}}{F})]\};$$

$$\begin{aligned}\overset{(s)}{\Pi}{}^{.m}_{nm}\Gamma^n_{il}g^{il}=\frac{1}{4}\frac{1}{(B^2+AC)^2}&[-A^2C^nC_n-4B^2B^nB_n-C^2A_nA^n-4CBA_nB^n\\&-2ACA_nC^n-4ABB_nC^n]\\&+\frac{1}{B^2+AC}\Big[-\frac{2A}{D}C^nD_n-\frac{4B}{D}B_nD^n-\frac{2C}{D}A_nD^n-\frac{2A}{F}C_nF^n\\&-\frac{4B}{F}B_nF^n-\frac{2C}{F}A_nF^n-2CA_ns^nF_ls^l-2CA_np^nD_lp^l\\&-4BB_np^nD_lp^l-4BB_ns^nF_ls^l-2AC_np^nD_lp^l-2AC_ns^nF_ls^l\Big]\end{aligned} \tag{10}$$
$$-\frac{2(D_np^n)^2}{D}-\frac{2(F_ns^n)^2}{F}-\frac{2}{D}D_ns^nF_ls^l-\frac{2}{DF}D_nF^n-\frac{2}{F}D_lp^lF_np^n-\frac{1}{D^2}D_nD^n-\frac{1}{F^2}F^nF_n.$$

$$\overset{(s)}{\Pi}{}^{.m}_{nm}\Gamma^n_{il}g^{il}=\frac{1}{4}\{\frac{1}{(B^2+AC)^2}[A^2(\frac{C^{*2}}{D}+\frac{C^{\Delta^2}}{F})+4B^2(\frac{B^{*2}}{D}+\frac{B^{\Delta^2}}{F})+C^2(\frac{A^{*2}}{D}+\frac{A^{\Delta^2}}{F})+4CB(\frac{A^*B^*}{D}+\frac{A^{\Delta}B^{\Delta}}{F})+2AC(\frac{A^*C^*}{D}+\frac{A^{\Delta}C^{\Delta}}{F})+4AB(\frac{B^*C^*}{D}+\frac{B^{\Delta}C^{\Delta}}{F})]+\frac{1}{B^2+AC}[\frac{2A}{D}(\frac{C^*D^*}{D}+\frac{C^{\Delta}D^{\Delta}}{F})+\frac{4B}{D}(\frac{B^*D^*}{D}+\frac{B^{\Delta}D^{\Delta}}{F})+\frac{2C}{D}\left(\frac{A^*D^*}{D}+\frac{A^{\Delta}D^{\Delta}}{F}\right)+\frac{2A}{F}\left(\frac{C^*E^*}{D}+\frac{C^{\Delta}F^{\Delta}}{F}\right)+\frac{4B}{F}\left(\frac{B^*F^*}{D}+\frac{B^{\Delta}F^{\Delta}}{F}\right)+\frac{2C}{F}(\frac{A^*F^*}{D}+\frac{A^{\Delta}F^{\Delta}}{F})-\frac{2CA^{\Delta}F^{\Delta}}{F^2}-\frac{2CA^*D^*}{D^2}-\frac{4BB^*D^*}{D^2}-\frac{4BB^{\Delta}F^{\Delta}}{F^2}-\frac{2AC^*D^*}{D^2}-\frac{2AC^{\Delta}F^{\Delta}}{F^2}]-\frac{2D^{*2}}{D^3}-\frac{2F^{\Delta^2}}{F^3}+\frac{1}{D^2}(\frac{D^{*2}}{D}+\frac{D^{\Delta^2}}{F})+\frac{1}{F^2}(\frac{F^{*2}}{D}+\frac{F^{\Delta^2}}{F})\}$$

Taking into account the results (9) and (10), the following was obtained:

$$L = \frac{1}{2DF}\{-\frac{1}{(B^2+AC)^2}\left[B^2(FA^*C^* + DA^{\Delta}C^{\Delta}) + 2B^2\left(FB^{*2} + DB^{\Delta 2}\right) + AC(FA^*C^* + DA^{\Delta}C^{\Delta})\right] + \frac{1}{(B^2+AC)}\left[AC^{\Delta}D^{\Delta} + 2BB^{\Delta}D^{\Delta} + CA^{\Delta}D^{\Delta} + AC^*F^* + 2BB^*F^* + CA^*F^* - FB^{*2} - DB^{\Delta 2}\right] + \frac{FD^{*2}}{D^2} + \frac{DF^{\Delta 2}}{F^2}\} \quad (11)$$

The symbols * and Δ mean differentiation with respect to the coordinates p and q, respectively. The structure functions in the limit of flat space-time [22] were represented as:

$$P(p) = \frac{a}{s^2a^2+1}\left[a(1-p^4) - \frac{(s^2a^2-1)}{s}p^2\right]$$
$$Q(q) = \frac{a}{s^2a^2+1}\left[a(1-q^4) + \frac{(s^2a^2-1)}{s}q^2\right] \quad (12)$$

The parameter $s$ is related to the rotation of space-time. Parameter $a$ has the value of the acceleration.

Taking into account (12), functions (6) will take the form:

$$A = \frac{a\{as[1-q^4-q^2(1-p^4)]+(s^2a^2-1)(1+p^2)q^2\}}{s(s^2a^2+1)(p+q)^2(1+(pq)^2)}$$
$$B = \frac{2a^2\{p^2(1-q^4)+q^2(1-p^4)\}}{(s^2a^2+1)(p+q)^2(1+(pq)^2)}$$
$$C = -\frac{a\{sa[\,p^4(2-q^4)-1]+(s^2a^2-1)(1+q^2p^2)p^2\}}{s(s^2a^2+1)(p+q)^2(1+(pq)^2)}$$
$$D = \frac{s(1+(pq)^2)(s^2a^2+1)}{a[sa(1-p^4)-(a^2s^2-1)p^2](p+q)^2}$$
$$F = \frac{s(s^2a^2+1)(1+(pq)^2)}{a(p+q)^2[sa(1-q^4)+(a^2s^2-1)q^2]}$$

**Predictive modeling of critical phenomena in island systems.** Predictive modeling of critical phenomena of system (1) for the case a>0 and s>0 was carried out [22]. The position of the points in space at which the stability condition is satisfied was calculated from the system [20, 21]:

$$\left|\vec{\nabla}L\right| = 0; \quad det\frac{d^2L}{dX^2} > 0, \quad (13)$$

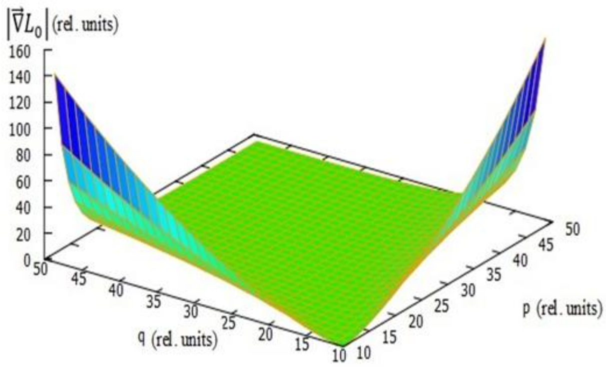


**Fig.1a.** Results of modeling the surface of the Lagrangian gradient modulus $\left|\vec{\nabla}L\right|$ in relative units

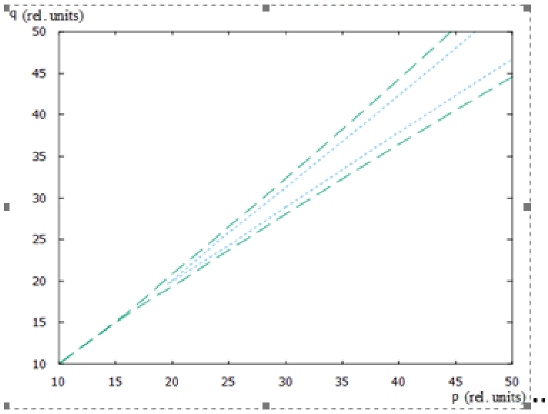


**Fig.1b.** Results of modeling the zero contour of the Lagrangian gradient modulus $|\vec{\nabla}L|$ in relative units

$$X = X(p;q), \qquad det\frac{d^2L}{dX^2} = det\begin{pmatrix} \frac{\partial^2(p,q)}{\partial p^2} & \frac{\partial^2 L(p,q)}{\partial p \partial q} \\ \frac{\partial^2 L(p,q)}{\partial q \partial p} & \frac{\partial^2 L(p,q)}{\partial q^2} \end{pmatrix}$$

The results of modeling the surface of the Lagrangian gradient modulus $|\vec{\nabla}L|$ and its zero contour in relative units are shown in Fig.1a and Fig.1b, respectively.

The results of modeling the surface of the determinant $det\frac{d^2L}{dX^2}$ of the second derivatives of the Lagrangian and its zero contour in relative units are shown in Fig.2a and Fig.2b, respectively.

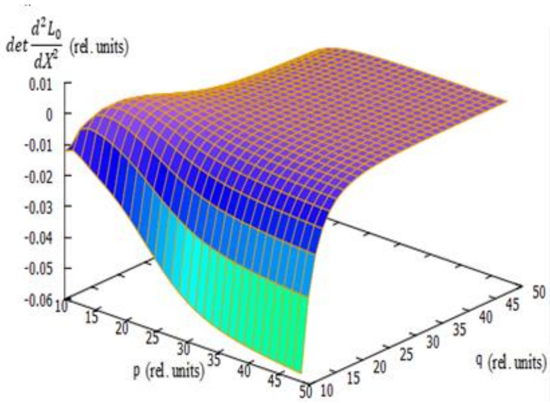


**Fig.2a.** Results of modeling the surface of the determinant of the second derivatives of the Lagrangian in relative units.

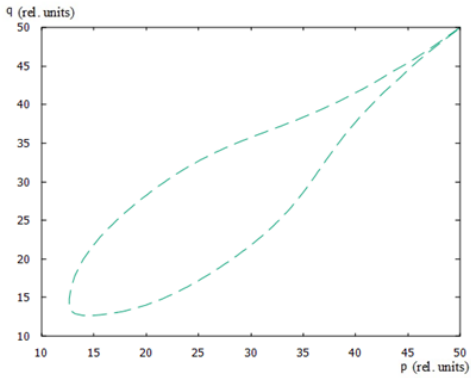


**Fig.2b.** Results of modeling the zero contour of the determinant of second derivatives in relative units

The position of the points at which the conditions for the emergence of the bifurcation space of the studied model (1) are fulfilled was calculated from the system of equations [22, 23]:

$$\left|\vec{\nabla}L\right| = 0; \quad det\frac{d^2L}{d\mathrm{X}^2} = 0 \tag{14}$$

The zero contour of the determinant of the second derivatives ($det\frac{d^2L}{d\mathrm{X}^2} = 0$) of the Lagrangian (9) is shown in Fig. 2b. The position of the points on the section of the phase diagram, in which the conditions for the emergence of a space of coexistence of two phases are fulfilled, was calculated from a system of equations and inequalities [22, 23]:

$$\left|\vec{\nabla}L\right| = 0, \quad det\frac{d^2L}{d\mathrm{X}^2} = 0,\ det\frac{d^3L}{d\mathrm{X}^3} = 0,\ det\frac{d^4L}{d\mathrm{X}^4} > 0 \tag{15}$$

where $det\frac{d^3L}{d\mathrm{X}^3}$ is the determinant of the third derivative of the Lagrangian with respect to the arguments p and q. The block matrix diagonalization algorithm was used to obtain the matrix of third derivatives of the Lagrangian. Analytical expressions of the third partial derivatives of the Lagrangian were obtained in the first step. Two matrices of partial derivatives of the components of the matrix of second derivatives of the Lagrangian with respect to arguments $p$ and $q$ were formed, respectively:

$$\frac{d^3L}{dp^3} = \begin{pmatrix} \frac{\partial^3 L(p,q)}{\partial p^3} & \frac{\partial^3 L(p,q)}{\partial p\partial q\partial p} \\ \frac{\partial^3 L(p,q)}{\partial q\partial p\partial p} & \frac{\partial^3 L(p,q)}{\partial q^2\partial p} \end{pmatrix}, \quad \frac{d^3L}{dq^3} = \begin{pmatrix} \frac{\partial^3 L_0(p,q)}{\partial p^2\partial q} & \frac{\partial^3 L_0(p,q)}{\partial p\partial q\partial q} \\ \frac{\partial^3 L_0(p,q)}{\partial q\partial p\partial q} & \frac{\partial^3 L_0(p,q)}{\partial q^3} \end{pmatrix} \tag{16}$$

The block-diagonal matrix of the third derivative of the Lagrangian of system (1) with respect to arguments p and q was obtained from (16) in the next step:

$$\frac{d^3L}{d\mathrm{X}^3} = \begin{pmatrix} \frac{d^3L}{dp^3} & 0 \\ 0 & \frac{d^3L}{dq^3} \end{pmatrix} \tag{17}$$

The results of calculating the positions of the points of the surface of the determinant of the third derivative of the Lagrangian (9) and its zero contours are shown in Fig. 3a and Fig. 3b, respectively.

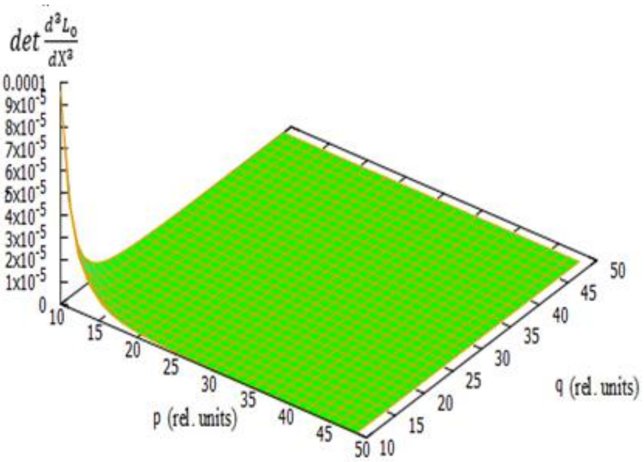


**Fig. 3a.** Results of calculating the surface of the determinant of the third derivative of the Lagrangian.

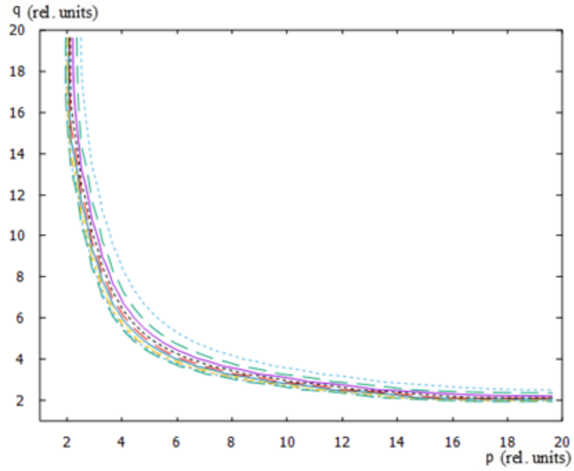


**Fig. 3b.** Results of calculation of zero contours of the determinant of the third derivative of the Lagrangian.

The calculated surface of the determinant of the fourth derivatives of the Lagrangian (9) on the studied interval is shown in Fig.4.

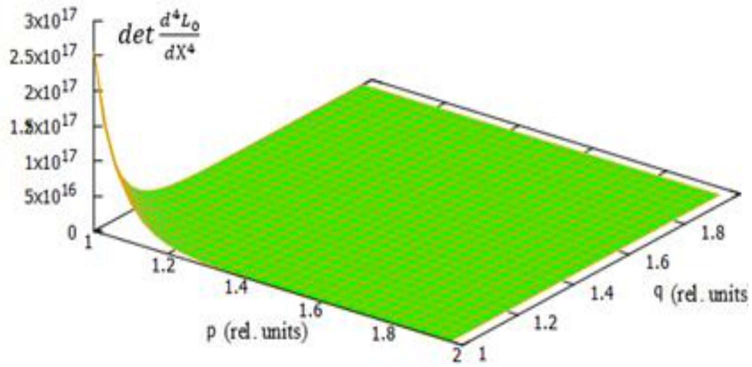


**Fig. 4.** Results of calculating the position of the points of the surface of the determinant of the fourth derivative of the Lagrangian in relative units

Analytical expressions of the fourth partial derivatives of the Lagrangian (1) with respect to $X(p,q)$ for calculating the determinant of the fourth derivative were obtained. Matrices of partial derivatives in the next step were composed:

$$\frac{d^4L}{dp^4} = \begin{pmatrix} \frac{\partial^4 L(p,q)}{\partial p^4} & \frac{\partial^4 L(p,q)}{\partial p \partial q \partial p^2} \\ \frac{\partial^4 L(p,q)}{\partial q \partial p^3} & \frac{\partial^4 L(p,q)}{\partial q^2 \partial p^2} \end{pmatrix}; \quad \frac{d^4L}{dp^3 q} = \begin{pmatrix} \frac{\partial^4 L(p,q)}{\partial p^3 \partial q} & \frac{\partial^4 L(p,q)}{\partial p \partial q \partial p \partial q} \\ \frac{\partial^4 L(p,q)}{\partial q \partial p^2 \partial q} & \frac{\partial^4 L(p,q)}{\partial q^2 \partial p \partial q} \end{pmatrix}$$

$$\frac{d^4L}{dq^3 p} = \begin{pmatrix} \frac{\partial^4 L_0(p,q)}{\partial p^2 \partial q \partial p} & \frac{\partial^4 L_0(p,q)}{\partial p \partial q^2 \partial p} \\ \frac{\partial^4 L_0(p,q)}{\partial q \partial p \partial q \partial p} & \frac{\partial^4 L_0(p,q)}{\partial q^3 \partial p} \end{pmatrix}; \frac{d^4L}{dq^4} = \begin{pmatrix} \frac{\partial^4 L_0(p,q)}{\partial p^2 \partial q^2} & \frac{\partial^4 L_0(p,q)}{\partial p \partial q^3} \\ \frac{\partial^4 L_0(p,q)}{\partial q \partial p \partial q^2} & \frac{\partial^4 L_0(p,q)}{\partial q^4} \end{pmatrix}.$$

The block-diagonal matrix $\frac{d^4L_0}{dX^4}$ was composed from the obtained matrices at the next step:

$$\frac{d^4L_0}{dX^4} = \begin{pmatrix} \frac{d^4L}{dp^4} & 0 & 0 & 0 \\ 0 & \frac{d^4L}{dp^3q} & 0 & 0 \\ 0 & 0 & \frac{d^4L}{dq^3p} & 0 \\ 0 & 0 & 0 & \frac{d^4L}{dq^4} \end{pmatrix} \quad (18)$$

Analysis of the position of the points of the surface of the determinant of the fourth derivative of the Lagrangian (9) of the system under study (1) showed that the condition $det\frac{d^4L_0}{dX^4} > 0$, that is, the positive signature of the calculated points of the surface of the determinant of the fourth derivative of the Lagrangian, is satisfied over the entire range of the phase space under study. The differential-topological method for calculating the position of the points of the phase space of system (1) in which the conditions for the emergence of phase coexistence spaces of order two are satisfied was applied. The positions of the points on the phase space at which conditions (15) are simultaneously satisfied in the region under study were determined. Analytical expressions for first- to fourth-order derivatives and calculations of the position of the surface points of the determinants of the corresponding derivatives using the open computer algebra system MAXIMA [24] were determined. The positions of the points on the section of the phase diagram of the existence of the system (1), in which the condition of coexistence of second-order phases (15) is fulfilled, are shown in Fig. 6. The conditions of zero values R of the derivatives from the first to the third inclusive and positive values of the fourth derivative of the Lagrangian (9) of the system under study (1) at these points are fulfilled simultaneously. The points in the phase space where the two phases coexist are located along the boundary of the found region (Fig. 6). Condition (15) is fulfilled along the boundary of the found space. Thus, the simulation results predict the emergence of spaces of coexistence of two phases near the found boundary. System (1) will be in different phases on both sides of the found boundary.

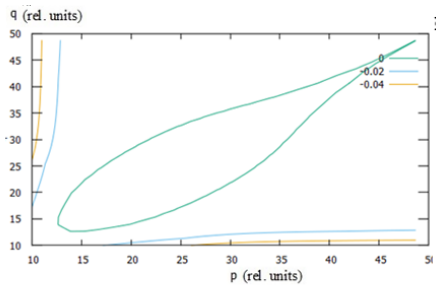


**Fig. 6.** Results of calculations of the section of the phase diagram of the system (1). The positions of the points at which the condition of coexistence of second-order phases is fulfilled are shown (in relative coordinates).

**Conclusions.** The results of modeling critical phenomena in island systems of the Plebanski-Demianski type indicate the existence of a space in which the stability condition of the system under consideration is met, as well as the possibility of the emergence of bifurcation regions under certain conditions. Calculations of sections of phase diagrams of the considered system show that at certain values of parameters and independent coordinates of the model, the conditions for the emergence of phase coexistence spaces of order two will be fulfilled. Such states are unstable and can lead to the degradation of the system by a jump. The relativity of coordinates allows us to adapt the proposed approach to predict the emergence of critical spaces on phase sections of existence diagrams of both various elementary particles and island macrosystems.

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## МОДЕЛЮВАННЯ КРИТИЧНИХ ЯВИЩ В ОСТРІВНИХ СИСТЕМАХ ПЛЕБАНСЬКОГО-ДЕМ'ЯНСЬКОГО З УРАХУВАННЯМ ОБЕРТАННЯ ТА ПРИСКОРЕННЯ

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Математичне моделювання критичних явищ у системах, що описуються острівними моделями, є актуальним. Це пов'язано з тим, що острівні моделі можуть бути використані для прогнозування нестійких станів елементарних частинок, що розширює можливості прогнозування процесів у сучасній ядерній енергетиці. Можна виконати моделювання фазового переходу вакуум-матерія на основі аналізу критичних явищ в острівних системах. Основні положення теорії катастроф Тома та фазових переходів Ландау були використані для прогнозування можливості виникнення критичних явищ в острівних системах. Розрахунки фазових станів були виконані з використанням диференціально-топологічного підходу. Результати розрахунків вказують на можливість виконання умов фазових переходів другого роду, коли дві різні фази системи можуть співіснувати одночасно. Система стає нестійкою та може переходити з однієї стабільної фази в іншу навіть з невеликими коливаннями за таких умов. Для розрахунку фазових станів острівних систем було досліджено модель острова Плебанського-Дем'янського з урахуванням обертання та прискорення.

**Ключові слова:** співіснування фаз, критичні явища, фазові простори, острівні системи, матричний визначник.