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NON-CLASSICAL METHOD OF CALCULATING THE INTEGRAL COMPONENT IN REGULATORS OF MULTIVARIABLE DISCRETE-TIME CONTROL SYSTEMS

O.A. Stopakevych[1], A.O. Stopakevych[2]

[1] National Odesa Polytechnic University
1 Shevchenko Ave., Odesa, 65044, Ukraine
[2] State University of Intellectual Technologies and Telecommunications
1, Kuznechna, 65000, Odesa, Ukraine
e-mail: stopakevich@gmail.com

A new method of designing controllers with an integral term as part of multivariable optimal control systems in discrete time is proposed. Traditional methods of designing multivariable controllers have been investigated, and a prototype of the controller has been chosen as a basis for comparison. Specific formulas for design of the proposed controller are given. Comparing the transition processes of the control systems outputs with a unit-step disturbance at the plant input using the proposed controller and optimal controller obtained by the typical method, it can be concluded that the proposed controller compensates the disturbance and gives a faster transition process. The main feature of the proposed method is that, for multivariable systems with a large number of inputs and outputs, it significantly simplifies the design of the controller. This is because it does not require the extension of the plant state matrix by the number of inputs at the transfer of the matrices to the optimal controller design programs.
Keywords. MIMO optimal controller, integral term, new design method, computational simplification of design.

Analysis of the problem. It is known that the integral component in regulators performs an important function in ensuring the accuracy of reference maintenance and compensation of disturbances in control systems. Let us briefly consider the implementation options of the integral component in multidimensional optimal control systems in discrete time.

Let's define a multidimensional control plant in the state space in the form

$$x_{i+1} = A \cdot x_i + B \cdot u_i + B \cdot f_i$$
$$y_i = C \cdot x_i$$

We'll assume that the plant under consideration has the same number of inputs and outputs.

Let's consider the main options for including integrators in the controllers of multivariable discrete control systems [1-5] (Fig. 1).

The simplest is option **1**. Here, a multivariable discrete integrator is simply included in parallel with the state controller. Accordingly, it is optimal according to the integral criterion

$$J = \frac{1}{2}\sum_{i=0}^{\infty} x_i^T \cdot Q \cdot x_i + u_i^T \cdot R \cdot u_i$$

MIMO state controller can be designed with the MATLAB function C1=dlqr(Ad,Bd,Q,R). Accordingly, the optimal multivariable state controller is calculated using the MATLAB program Ad1=[Ad zeros(n,m); C eye(m)]; Bd1=[Bd; zeros(m)]; [Kd,P]=dlqr(Ad1,Bd1,Q,R); C1=Kd(m:n);C2=Kd(m, n+m);

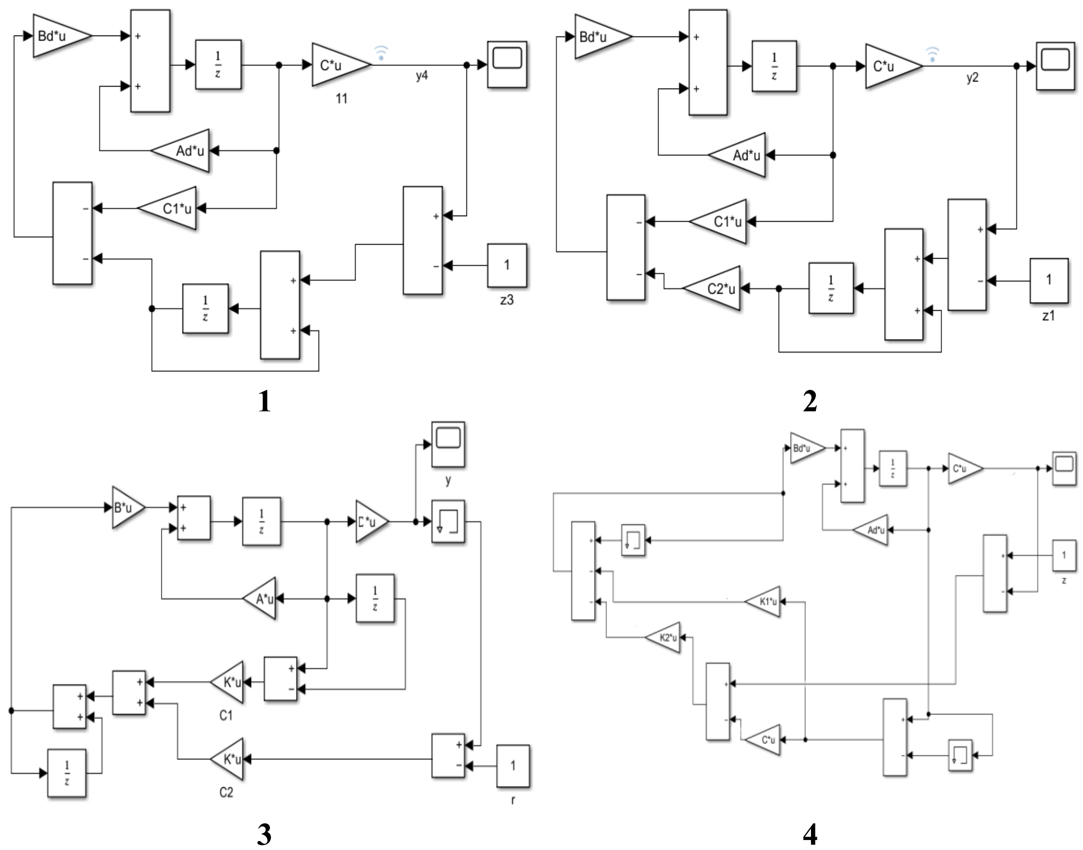

Fig 1. Diagrams of the main options of integral action realization in discrete MIMO control systems

The main options for including integrators in the controllers of multivariable discrete systems.

In option **2**, the parameters of the I-component are determined by the extended matrix

$$\begin{bmatrix} x_{i+1} \\ q_{i+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & I \end{bmatrix} \cdot \begin{bmatrix} x_i \\ q_i \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \cdot u_i + \begin{bmatrix} B \\ 0 \end{bmatrix} \cdot f_i$$

In option **3**, the control is implemented in the velocity or differential form. The control law has the form (r is the reference)

$$u_i = u_{i-1} + C_1 \cdot x_i + C_2 \cdot (y_{i-1} - r).$$

This implementation is less accepted, but it has one advantage, namely there is the ability to limit the rate of change of state variables and control actions.

Option **4** is quite effective, but structurally more complex.

Let's introduce optimality criterion in the form:

$$J = \frac{1}{2} \sum_{i=0}^{\infty} e_i^T \cdot Q \cdot e_i + \Delta u_i^T \cdot R \cdot \Delta u_i,$$

where $\Delta u_i = u_i - u_{i-1}, e_i = r - y_i$

Determining , $\Delta x_{i+1} = x_{i+1} - x_i$, the system can be written in the form

$$\Delta x_{i+1} = A \cdot \Delta x_i + B \cdot \Delta u_i$$

$$\Delta y_i = C \cdot \Delta x_i$$

By entering the vector $e_{i+1} = e_i - C \cdot \Delta x_{i+1}$, we will get the standard problem of controller design

$$K = (K_1, K_2) = dlqr(A1, B1, Q1, R),$$

$$A1 = \begin{pmatrix} A & 0 \\ -C \cdot A & I \end{pmatrix}, \quad B1 = \begin{pmatrix} B \\ -C \cdot B \end{pmatrix}, \quad Q1 = \begin{pmatrix} 0 & 0 \\ 0 & Q \end{pmatrix}$$

Thus, the controller can be written as $\Delta u_i = -K \cdot \Delta x_i$. Decomposing the matrix *K* into blocks *K=[K1 K2],* the controller can be rewritten in the form $u_i = u_{i-1} - K_1 \cdot \Delta x_i - K_2 \cdot e_i$ or, which is also the same,

$$u_i = -K_1 \cdot x_i - K_2 \cdot \sum_{j=0}^{i} e_j$$

The program code for the controller matrix calculation has the following form:

```
A1=[A zeros(n,m); -C*A eye(m)];
B1=[B;-C*B];
Q1=[zeros(m,n+m); zeros(n,m) Q];
K=dlqr(A1,B1,Q1,R);
K1=K(m:n);   K2=K(m,n+1:m+n);
```

Thus, the control law for the system can be written as $\Delta u_i = -K \cdot \Delta x_i$. Decomposing the matrix *K* into blocks *K=[K1 K2]*, the controller can be rewritten in the form or, which is also the same, $u_i = u_{i-1} - K_1 \cdot \Delta x_i - K_2 \cdot e_i$

In option **4,** the control problem is reduced to finding four unknown matrices K, L, M, N

$$L \cdot B \cdot R \cdot B^T \cdot M - Q = 0;$$

$$(L \cdot B \cdot R \cdot B^T \cdot N) + K + L \cdot A = 0;$$

$$-(N \cdot B \cdot R \cdot B^T \cdot M) + K + A^T \cdot M = 0;$$

$$-(N \cdot B \cdot R \cdot B^T \cdot N) + M + N \cdot A + L + A^T \cdot N = 0$$

To solve the system it is necessary to use optimization algorithms. For example, the YALMIP library allows to conveniently solve optimization problems and is focused on control system design. This library works in both MATLAB and Octave. The software implementation of the optimization problem has the following form.

```
A=[-0.313 56.7 0;-0.0139 -0.426 0;0 56.7 0];
B=[0.232;0.0203;0];C=[0 0 1]; D=0;
Q=1diag([1 1 5e2]); R=1/10; % inv(R)
L=sdpvar(3,3,'full'); K=sdpvar(3,3,'full');
M=sdpvar(3,3,'full'); N=sdpvar(3,3,'full');
eps=0.01;
Constraints = [0<=y1<=eps;0<=y2<=eps,0<=y3<=eps;0<=y4<=eps;];
sol = optimize(Constraints);
N1=double(N);M1=double(M);
KP=inv(R)B'N1; KI=inv(R)B'*M1;
```

It should be noted that the solution of even a relatively simple problem is not trivial. The problem is not solved for all possible *R* and *Q*. There is a fairly high chance of obtaining an unstable system due to the presence of an imprecise solution. Since, as a rule, in order to achieve a significantly different result, the weight matrices coefficients should be significantlly changed, the greater the difference between the coefficients, the less accurate the solution. Therefore, it is quite difficult to apply the given option as a universal one. Nevertheless, this option has one advantage: the integral component is found based on the criterion of optimality.

To compare the action of the considered options, we will simulate them. To do this, let's set the object model in the form with one input and one output (to simplify the analysis of responces) in the form Ac=[-0.313 56.7 0; -0.0139 -0.426 0; 0 56.7 0]; Bc=[0.232;0.0203;0]; C=[0 0 1]; D=0; dt=1; [A B]=c2d(Ac,Bc,dt); Q=diag([1 1 1]); R=1. Graphs of responces are shown in Fig. 2.

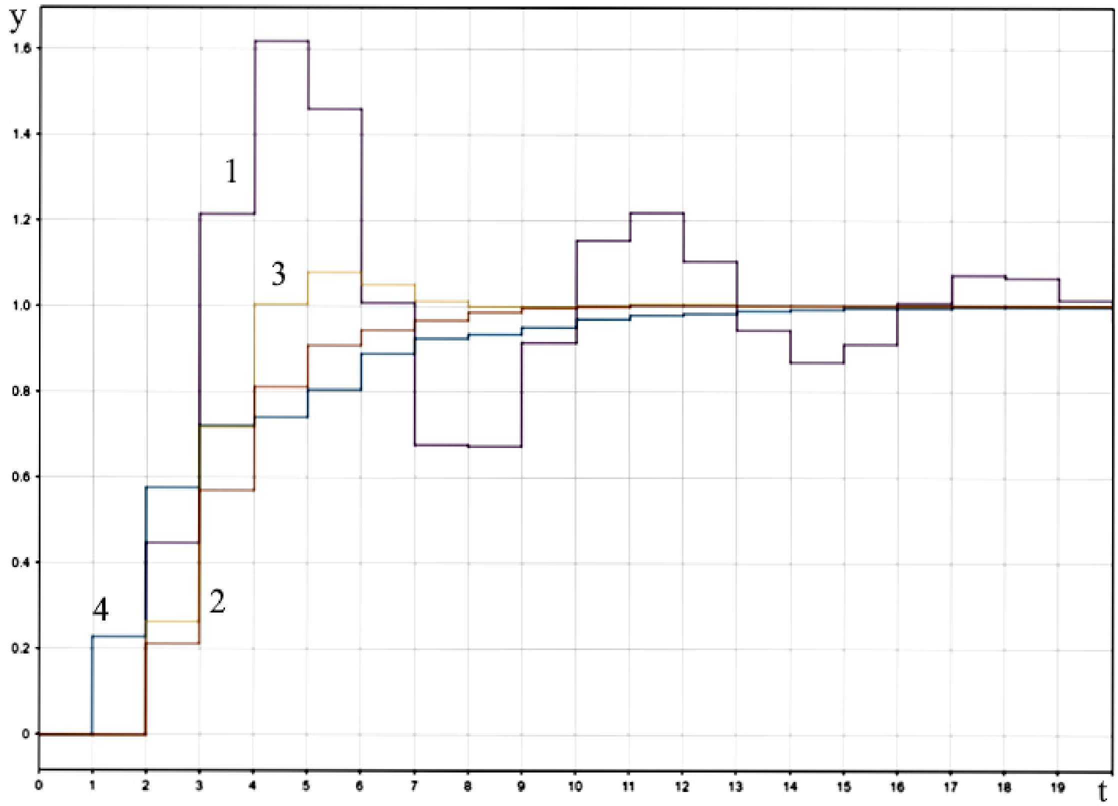

Fig. 2. Comparison of the closed-loop control systems simulation results using controllers of
options 1 –4

The figure shows that the use of a simple integrator (option 1) significantly reduces the control quality. Other options in the given example give approximately the same results. Therefore, for further research, we will choose the typical (simpler) option 2. **Main part.** Instead of the typical state space expansion procedure used in the description of option 2, we offer a modified procedure: synthesize a standard optimal state controller *K*, and then find the matrix of the integral component *KK* according to the formula

$$KK = \begin{pmatrix} K \cdot A - W \cdot B^T & W + K \cdot B - I \end{pmatrix} \cdot \begin{pmatrix} A - I & B \\ C & 0 \end{pmatrix}^{-1},$$

where *W* is a positive definite diagonal tuning matrix. The controller is given by the formula

$$u_{i+1} = u_i - K1 \cdot (x_{i-1} - x_i) - K2 \cdot y_i$$
$$K1 = KK(m,1:n), K2 = KK(m,n+1:n+m)$$

To study the transition processes with the proposed controller, we will choose a simple plant in discrete time, which in the state space is defined by matrices with the sample time dt=1.

$$A=\begin{pmatrix}0.5 & 0.2\\0.1 & 0.4\end{pmatrix}, B=\begin{pmatrix}1\\2\end{pmatrix}, C=\begin{pmatrix}3 & 4\end{pmatrix}$$

In the form of a transfer function, the plant has the form

$$W(z)=\frac{11\cdot z-3.6}{z^2-0.9\cdot z+0.18}$$

The plant step response is shown in Fig. 3

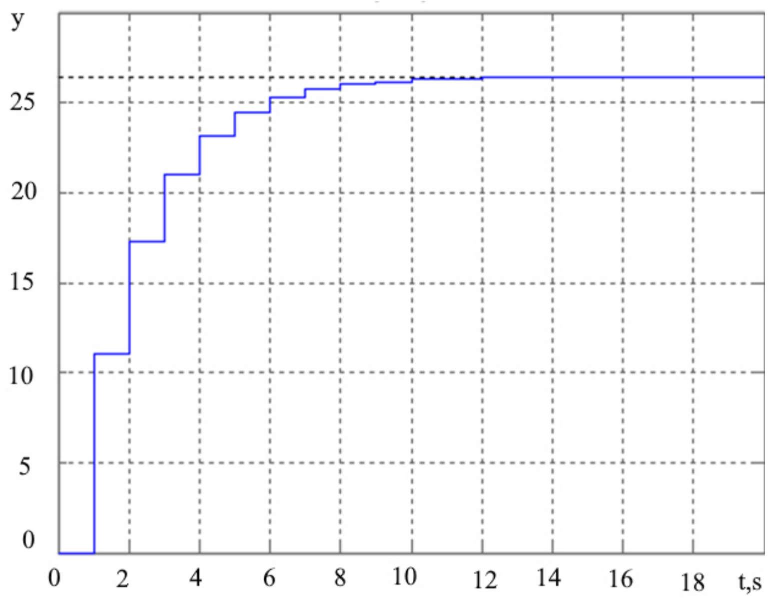

Fig. 3. The plant step response

Let's design the proposed controller by choosing
Q=1000*eye(2), R=1, W=0.14;
K=dlqr(A,B,Q,R);
KK=[K*A-W*B' W+K*B-1]*inv([A-eye(2) B; C 0]);

As a result, we get

KK=(-0.1816 -0.0628 -0.0427), K1=(-0.1816 -0.0628), K2=-0.0427.

The transient processes of output and control in a closed-loop control system with the proposed controller are shown in Fig. 4.

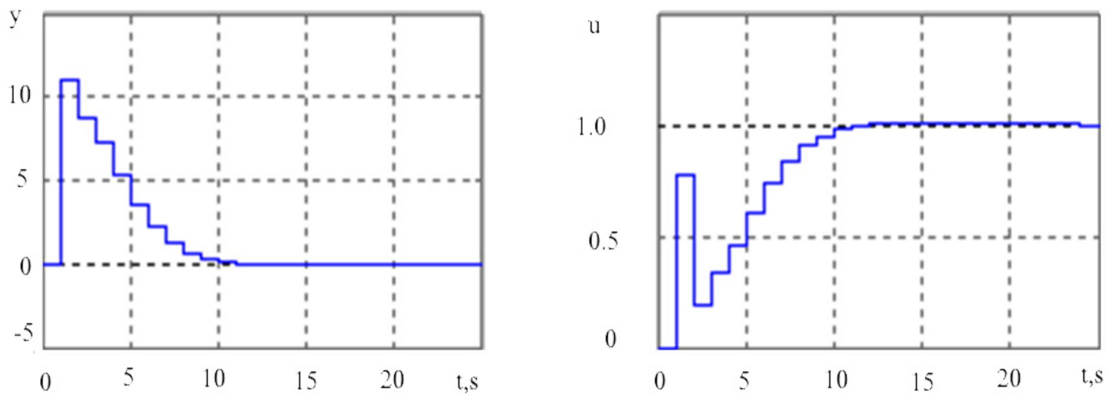

Fig. 4. Transient processes of output and control under a step disturbance in a closed-loop control system with the proposed controller

For comparison, Fig. 5 shows transient processes with the initial optimal static controller *K*.

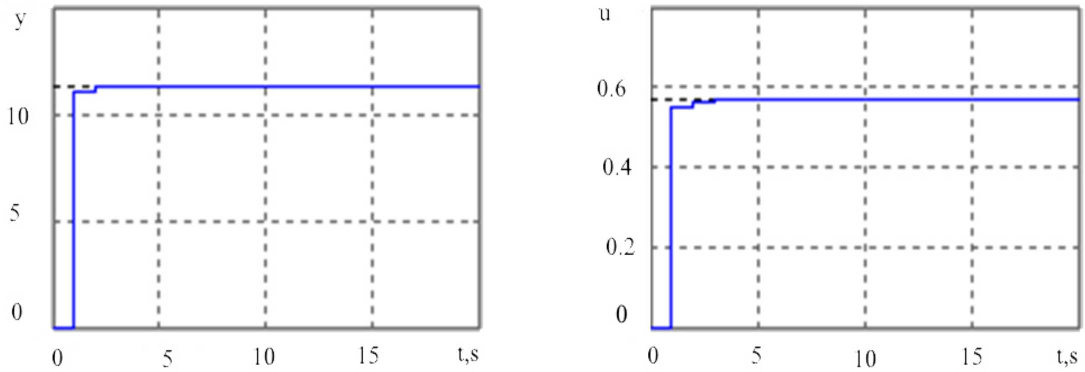

Fig. 5. The output transient processes in the closed-loop control system with the optimal static controller

Conclusions. To draw conclusions, we will compare the obtained responses with the responses in the system that uses a typical controller of option 2. For this, we will use the program code

```
AI=[A zeros(2,1);C 1]; BI=[B;0];
QI=[1000 0 0; 0 1000 0; 0 0 1]; RI=1;
KI=dlqr(AI,BI,QI,RI);
KI1=KI(1:2); KI2=KI(3);
```

We get KI= – (0.1954 0.2513 0.0128), KI1=-(0.1954 0.2513); KI2=–0.0128.

From the comparison of the responses in the control system at the output when using the proposed (1) and typical (2) controllers, it can be concluded that the proposed controller qualitatively compensates for the disturbance and provides a faster response. At the same time, for MIMO systems with a large number of inputs and outputs, it significantly simplifies the controller design, as it does not require the expansion of the matrix of plant states by the number of inputs before transferring the matrices to the design program (for example, dlqr).

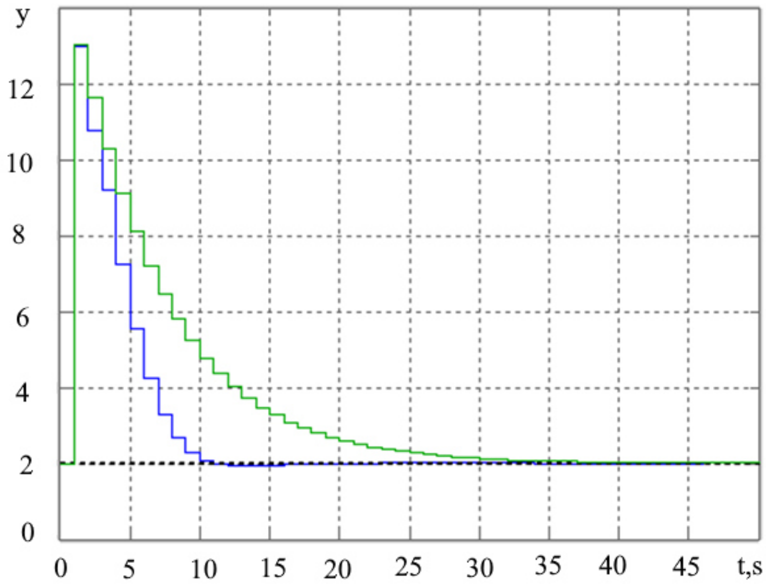

Fig. 6. Comparison of the closed-loop control systems simulation results using the proposed (1) and typical (2) controllers

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НЕКЛАСИЧНИЙ МЕТОД РОЗРАХУНКУ ІНТЕГРАЛЬНОЇ СКЛАДОВОЇ В РЕГУЛЯТОРАХ БАГАТОВИМІРНИХ СИСТЕМ УПРАВЛІННЯ В ДИСКРЕТНОМУ ЧАСІ

О.А. Стопакевич[1], А.О. Стопакевич[2]

[1] Національний університет «Одеська політехніка»
1, Шевченка пр., Одеса, 65044, Україна
[2] Державний університет інтелектуальних технологій і зв’язку
1, Кузнечна, Одеса, 65000, Україна
e-mail: stopakevich@gmail.com

Запропоновано новий метод розрахунку регуляторів з інтегральною складовою в складі багатовимірних оптимальних систем управління в дискретному часі. Проаналізовано традиційні методи розрахунку багатовиірних регуляторів, та обрано варіант регулятора як базу для порівняння. Приведені конкретні формули розрахунку запропонованого регулятора. З порівняння процесів в системі управління по виходу при одиничному збуренні по входу об’єкта при використанні запропонованого та синтезованого за типовим методом оптимальних регуляторів можна зробити висновок, що запропонований регулятор якісно компенсує збурення і дає більш швидкий процес. Основною особливістю запропонованого метода є то, що для багатовимірних систем з великою кількістю входів і виходів він суттєво спрощує обчислення регулятора, оскільки не потребує розширення матриці станів об’єкта на число входів при передачі матриць в програми синтезу оптимальних регуляторів.

Ключові слова. MIMO оптимальний регулятор, інтегральна складова, новий метод розрахунку, обчислювальне спрощення синтезу.