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RESEARCH OF SOLVABILITY OF TASK OF AUTHENTICATION OF WATER-OIL MIXTURES ON THE PARAMETERS OF TUNING OF MATHEMATICAL MODEL

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The formulation of the problem is set by the parametric identification for oil-water reservoirs in the case when one of the fluids being filtered is anomalous. Therewith, the identification problem is defined as an optimal control problem reduced to finding the extremum of the quality criterion (functional). The conditions of the existence and uniqueness of the solution to identify the mathematical model adjustment are obtained alongside with the differentiability of the quality criterion, and therefore the corresponding theorems are proved. Solving the problems of modelling and identifying anomalous diffusion processes is associated with a number of fundamental difficulties, both staging and computational. In this sense, it should be noted, in particular: the non-linear nature of the processes under study; the complexity of the geometry of the spatial modelling area and its boundaries; limitedness of the vector of measurements of the state space of the process and the number of points of application of control actions; high dimensions of the resulting finite-dimensional analogues of the mathematical model (MM). Mathematical methods for describing anomalous diffusion processes with a multiplicative representation of state functions, as well as for the case of multicomponent diffusing systems, have not received sufficient development. These problems require not only development, but also the development of new methods for studying anomalous diffusion processes. The pronounced direction of the development of anomalous diffusion processes determines the adequacy of their mathematical formalization based on the apparatus of variational inequalities.

Keywords: mathematical model, variational inequalities, mathematical method, modeling and identifying.

Introduction

The solution of the problem of modeling water-oil fields assumes that the values of the coefficients of the differential operators of these variational inequalities are known, which, in turn, are determined by the physical parameters of the modeling environment. For fluid mechanics problems, these are: porosity and permeability. However, when solving practical problems, quite often the values of these parameters are not known in advance, and, therefore, the functions describing them are not a priori set, or, in other words, the coefficients of the differential operators of the corresponding MM are not determined. This circumstance makes it necessary to formulate and solve the problems of identifying the parameters of the physical medium that precede the solution of the problems of controlling the processes under study (and if the coefficients of the initial MM are not completely determined, then the modeling problems).

It should be pointed out that the identified parameters can be not only functions of spatial coordinates, but also of the desired functions reservoir pressure and water saturation.

Note that the problems of identifying the parameters of the pore medium were previously solved for oil and oil and gas reservoirs (for example, [1, 3]. However, these problems were solved based on the assumption of ideal fluid filtration. In the case of filtration of liquids that do not obey the Darcy law, a number of important aspects arise that qualitatively change the formulation of the identification problem:

1. From the interaction with a porous medium having specific physical and chemical parameters, the filtering liquid can acquire an anomalous character, which requires the use of adequate MM when solving practical problems.
2. The result of solving the problem of modeling an oil reservoir can be the achievement of a limiting pressure gradient, which leads to the subsequent formulation and solution of the problem of determining stagnant zones, as well as their physical parameters.
3. In the case of multiphase filtration, which differs in that the filtering fluid only partially obeys the Darcy law (when a viscous fluid is displaced by a viscous fluid), the general identification problem is divided into particular problems of determining parameter fields for zones of predominant rheology of anomalous and viscous fluids, which, in the general case, they can have different formulations. In this case, it should also be taken into account that the space of states of a multiphase fluid is characterized by fields of two functions: and .
4. The water-driven regime of oil field development, when presented in the form of MM, is clearly non-linear, which, in turn, leads to a non-trivial formulation of the identification problem, when the desired parameter fields are found in the class of non-linear functions.

Purpose of the work

The purpose of the work is to obtain conditions for solving the problem of parametric identification of water-oil mixtures when presenting mathematical models of the latter in the form of variational inequalities.

Main part

In the practice of the geophysical research and oil production the spatial soil medium denotes the layer, which in addition to the geological components of different types of rocks, the horizons of groundwater and fluid minerals, in particular, also includes a number of technological components, such as production and injection wells etc. The reservoir porosity and permeability of its material should be considered as the most important geological characteristics. These characteristics determine, respectively, the relative share of the amount of space occupied by the rock itself, and the penetrating ability of the medium for the intrastratal fluid - phase to be infiltrated (filtrated) through it. It should be noted that the filtering intrastratal fluids in terms of the hydrodynamic theory can be regarded as viscous (ideal), obeying a linear Darcy law of motion, or as viscoplastic (abnormal) whose motion can not be described within the bounds of the mentioned law. [1] Viscoplasticity should be understood in terms of compressibility, which is specified by the oil complex fractional composition in particular. The most common technological mode of oil production is artificially created pressure by pumping water into the injection wells. This filtering is called viscoplastic rheology of viscoplastic (oil) and viscous (water) fluids [1]. The mathematical model (MM) of a physical process in the case of the collaborative filtering of viscoplastic and viscous fluids in the reservoir system can be represented as follows [2,3] (here and hereafter the index or parameters of summation will be denoted by $i, j, j_1, j_2, \ldots$ will be for the corresponding variables):

$$-\frac{m\partial S_2}{\partial t}(v - S_2) - \int_\Omega \sum_{i=1}^{n}\left[k_1 \frac{\partial^2 P}{\partial z_i^2}|v|\right]dz + \int_\Omega \sum_{i=1}^{n}\left[k_1 \frac{\partial^2 P}{\partial z_i^2}|S_2|\right]dz \geq \frac{1}{h}\sum_{j=1}^{K_1}\zeta_j(z)Q_{1_j}(t) \quad \forall v, S_2 \in K \tag{1}$$

$$-\frac{m\partial S_2}{\partial t} - \int_\Omega \sum_{i=1}^{n}\left(k_2 \frac{\partial^2 P}{\partial z_i^2}\right)dz = \frac{1}{h}\sum_{j=1}^{K_2}\zeta_j(z)Q_{2_j}(t) \tag{2}$$

$$P(0,z) = P_0(z) \quad S_2(0,z) = S_{2_0}(z) \tag{3}$$

$$\frac{\partial S_2(t,z)}{\partial \eta} \geq 0; \; S_2(t,z) < S_{2_{\max}} \tag{4}$$

$$\frac{\partial S_2(t,z)}{\partial \eta} = 0;\ S_2(t,z) \geq S_{2_{\max}} \tag{5}$$

where $P = P(t,z)$ — the distributed function of intratrastal pressure; $S_2 = S_2(t,z)$ — the distributed function of of water saturation; $v = v(t,z)$ — the distributed test function (with respect to the function of water saturation); $P_0(z)$, $S_{2_0}(z)$ — the initial values of the functions, of the intrastratal pressure and of water saturation respectively; $S_{2_{\max}}$ — the maximum value of water saturation; $k_1 = k_1(z)$, $k_2 = k_2(z)$ — the reservoir permeability of the material for the corresponding phase (index 1 — oil, index 2 — water); $m = m(z)$ — the porosity of the reservoir material; h — the bulk of reservoir rock; $Q_{1_j}(t)$, $Q_{2_j}(t)$ — the consumption function of the corresponding phases (debits); $\zeta_j(t)$ — the function determining the nature of fluid withdrawal from the j-th hole; K_1, K_2 — a number of production and injection wells, respectively; Ω — the spatial region where a physical process is developing; t — temporal value; z — spatial value; K — the functional space of the function definition for water saturation respectively; n — the number of spatial variables; η — normal to the boundary G of the spatial domain Ω.

Solving the direct problem of the research, i.e., tasks of modeling, filtration processes described by the system of the form (1) — (5) suggests that the values of coefficients of the differential operators for the corresponding expressions, defined by the physical parameters of the medium are known — in this case by the porosity $m(z)$ and permeability of the reservoir $k_l(z)$ $(l = 1,2)$. However, in practice, quite often the values of these parameters are not known and, therefore, the functions describing them are not specified a priori, and , or in other words, the coefficients the differential operators for the corresponding MM are not defined. The given circumstance conditions the necessity to formulate and solve the identification problems of the parameters in the physical environment (inverse problems) - the porosity and permeability of the preceding the solution for managing the process being investigated, and, if the coefficients of the original MM are not completely defined, then the modeling problems as well. Therewith, the porosity and permeability are the parameter settings for the MM of the physical process studied.

The problems of identification of the parameters for the reservoir system have been, for example, earlier solved for oil and oil-gas reservoir [4]. However, their decision was made on the assumption of ideal filtering liquids. In case of abnormal fluid filtration, a number of important aspects qualitatively change the problem of identification:

— interacting with a porous medium, with specific physical and chemical parameters the filtering fluid can acquire anomalous character that requires the use of adequate MM for solving practical problems;

— the result of solving the problem for water-oil reservoir simulation can be considered when the intrastratal pressure achieves the limiting gradient that leads to the subsequent formulation and solution of problem of determining the therein dead zones, as well as the problem of identification of the physical parameters in the reservoir;

— in case of the multiphase filtration the filterable mixture of anomalous and ideal fluids only partially obeys Darcy's law: for example, when displacing viscoplastic fluid with viscous fluid, the general problem of identification is divided into individual tasks of determining the zones of the parameter fields in preferential rheology of anomalous and viscous fluids that, in general, can have a different setting;

— MM of water drive in oil field development has a clearly pronounced non-linear character, which, in its turn, results in setting a non-trivial problem of identification and finds the required parameter fields in the class of nonlinear functions when being solved.

In what follows, the problem of identification of the filtration processes in porous media will refer to the determination of the fields of the porosity parameters $m(z)$ and permeability of the medium $k_l(z)\ (l=1,2)$ based on the results of measuring the intrastratal pressure $P(t,z)$ and flow rates $Q_j(t)$ in the system of wells which cover the reservoir.

The formalized statement of problem of identifying the anomalous fluids of the filtration processes in porous media as an optimization problem is offered. Let $m'(z)$ and $k_l'(z)\ (l=1,2)$ are the exact values of porosity parameters and permeability of the medium, respectively. For the j-th well in the time interval $t\in(0,t_k)$, the measured intrastratal pressure $P(t,z)$ of the filterable fluid is indicated through

$$F_j^P(t)=\int_{\Omega_j}P'(t,z)dz+\varepsilon_j^P(t),\ j=1,\dots,(K_1+K_2) \tag{6}$$

and water saturation $S_2(t,z)$ in the reservoir through

$$F_j^S(t)=\int_{\Omega_j}S_2{}'(t,z)dz+\varepsilon_j^S(t),\ j=1,\dots,(K_1+K_2) \tag{7}$$

where $P'(t,z)$, $S_2{}'(t,z)$ are the values of the intrastratal pressure and water saturation, determined in accordance with a mathematical model for the exact type (1) — (5) of the parameter values $m'(z)$ and $k_l'(z)\ (l=1,2)$; $\varepsilon_j^P(t)$ and $\varepsilon_j^S(t)$ — are respectively, the measurement error of the intrastratal pressure and water saturation in the j-th well. [4]

The functionals are introduced into consideration

$$J_1[m(z),k_1(z)]=\sum_{j=1}^{K_1+K_2}\left\{\int_{T_j}\left[P'(t,z_j,m,k_1)-F_j^P(t)\right]^2dt+\int_{T_j}\left[S_2{}'(t,z_j,m,k_1)-F_j^S(t)\right]^2dt\right\} \tag{8}$$

$$J_2[m(z),k_2(z)]=\sum_{j=1}^{K_1+K_2}\left\{\int_{T_j}\left[P'(t,z_j,m,k_2)-F_j^P(t)\right]^2dt+\int_{T_j}\left[S_2{}'(t,z_j,m,k_2)-F_j^S(t)\right]^2dt\right\} \tag{9}$$

where T_j is the period of time when the measurement $F_j^P(t)$ and $F_j^S(t)$, is done.

Since the exact values of the pressure $P'(t,z_j,m,k_1)$ and $P'(t,z_j,m,k_2)$, as well as the water saturation $S_2{}'(t,z_j,m,k_1)$ and $S_2{}'(t,z_j,m,k_2)$ included in the expressions (8), (9), are physically the same value i.e. mathematically $J_1=J_2$), then only one of the functionals, e.g. J_1, will be taken into account in the subsequent arguments.

One possible approach to the solution of formulated problem of identification is representing it in the form of an optimal control problem. Quality criterion for this can be a functional (8), and the problem itself in terms of the optimization will be as follows: to determine $\hat{m}(z)$, $\hat{k}_1(z)$ for which

$$J_1(\hat{m},\hat{k}_1)\le J_1(m,k_1)\ \forall(m(z),k_1(z))\in\Lambda_d, \tag{10}$$

where Λ_d is the admissible domain to determine the parameter fields $m(z)$, $k_1(z)$.

The aim to qualitatively analysis the problem of identification for water-oil reservoirs by the parameters of the MM settings in the work conducted is studying the existence and uniqueness of problem solving (10), as well as establishing the fact of differentiability of the functional $J_1[m(z),k_1(z)]$ in (8) by the of porosity and permeability parameters. In this regard, the following theorems are formulated and proved.

Theorem 1. For a set of functions defined by (6), (7) and the admissible domain of the parameters $\forall\Lambda_d^m, \Lambda_d^k \in \Lambda_d$, the problem (10) has, at least, one solution, and this solution is the only one.

Proof. Given the physical meaning of the operators and domain of admissible values of variables included in the system (1) — (5), their affiliation the corresponding class of spaces is written as

$$P(t,z) \in L^2(\Omega) = H(\Omega); \quad S_2(t,z) \in L^2(\Omega) = H(\Omega);$$

$$\frac{1}{h}\sum_{j=1}^{K_1} \zeta_j(z) Q_{1_j}(t) = f_1(t,z) \in L^2(\Omega); \quad \frac{1}{h}\sum_{j=1}^{K_{21}} \zeta_j(z) Q_{2_j}(t) = f_2(t,z) \in L^2(\Omega);$$

$$\int_\Omega \sum_{i=1}^{n} \left[k_1 \frac{\partial^2 P}{\partial z_i^2} |v| \right] dz = A_1'(P,v,t,z) \in L^2(\Omega);$$

$$\int_\Omega \sum_{i=1}^{n} \left[k_1 \frac{\partial^2 P}{\partial z_i^2} |S_2| \right] dz = A_1'(P,S_2,t,z) \in L^2(\Omega);$$

$$\sum_{i=1}^{n} \left(k_1 \frac{\partial^2 P}{\partial z_i^2} \right) = A_1''(P,t,z) \in L^2(\Omega),$$

where $L^2(\Omega)$ is space of square-integrable functions.

Let a given functional space is $W^p = H^1(\Omega)$; $W^S = H^1(\Omega)$; $H(\Omega) = L^2(\Omega)$, where $H^1(\Omega)$ is Sobolev space of order 1, defined as follows

$$H^1(\Omega) = \left\{ \omega \middle| \omega \in L^2(\Omega);\ \frac{\partial \omega}{\partial z_i} \in L^2(\Omega),\ i = 1,2 \right\}.$$

It is assumed that there are sets of elements in spaces W^P and W^S, which are generated by a basis for which the following relations are true

$$\left(\left(w_j^P, \omega\right)\right) = \beta_j^P \left(w_j^P, \omega\right)\ \forall w^P \in W^P;\ \forall j = 1,2,...,q,$$

$$\left(\left(w_j^S, \omega\right)\right) = \beta_j^S \left(w_j^S, \omega\right)\ \forall w^S \in W^S;\ \forall j = 1,2,...,q.$$

Since the original system (1) — (5) is infinite, which is impossible to obtain an analytical solution for, it is necessary to pass to a discrete space for its numerical implementation. Then for the discrete space $W : W_n = \{w_1, w_2, ..., w_n\}$ the system, defining the problem (1) — (5) is written

$$-\left(\frac{m\,\partial S_2}{\partial t}, w_j^S\right)\left(v - S_2, w_j^S\right) - A_1'\left(P_q, v, t, z, w_j^P\right) + A_1'\left(P_q, v, t, z, w_j^P\right) \ge f_1\left(z, w_j^P\right); \quad j = 1,2,...,q \tag{11}$$

$$\left(\frac{m\,\partial S_2}{\partial t}, w_j^S\right) - A_1''\left(P_q, t, z, w_j^P\right) = f_2\left(z, w_j^P\right);\ j = 1,2,...,q \tag{12}$$

$$\overline{P_q}(0) = \overline{P_{0_q}} \to P_0 \in L^2(\Omega);\ \overline{S_{2q}}(0) = \overline{S_{2_{0_q}}} \to S_{2_0} \in L^2(\Omega) \tag{13}$$

where the set $\{P_q(t,z), S_{2_q}(t,z)\}$ is the approximate solution of (1) — (5), represented as

$$\overline{P}_j(t,z) = \sum_{j=1}^{q} \beta_j^P(t) w_j^P;\ \overline{S_2}_j(t,z) = \sum_{j=1}^{q} \beta_j^S(t) w_j^S,\ \forall t \in [0, t_q];$$

$\beta_j^P(t)$ and $\beta_j^S(t)$ are the weighting coefficients

The resulting solution is local because it is valid only on the local interval $t \in [0, t_q]$, and t_q is a discrete analog of t_k. It should be proved that $t_q = t_k$, i.e., that the local solution can be extended to the whole-time interval $\forall t \in [0, t_k]$.

For this purpose, the termwise multiplication of the derivatives of j-th dynamics ratios (11), (12) by $\beta_j^P(t)$ and $\beta_j^S(t)$, respectively, as well as their summation is performed, and as result the system of equations has the form of

$$-\left(\frac{m\,\partial S_{2q}(t,z)}{\partial t}, S_{2q}(t,z)\right)\left(v - S_{2q}(t,z)\right) - A_1'\left(P_q(t,z), v, P_q(t,z)\right) + \\ + A_1'\left(P_q(t,z), S_{2q}(t,z), P_q(t,z)\right) \geq f_1\left(z, P_q(t,z)\right) \tag{14}$$

$$\left(\frac{m\,\partial S_{2q}(t,z)}{\partial t}, S_{2q}(t,z)\right) - A_1''\left(P_q(t,z), P_q(t,z)\right) = f_2\left(z, P_q(t,z)\right) \tag{15}$$

$$P_q(0) = P_0;\ S_{2_q}(0) = S_{2_0} \tag{16}$$

where it turns out that the solution of $\{P(t,z),\ S_2(t,z)\}$ systems (1) — (5) exist in the whole interval $[0,t_k]$, i.e. $t_q = t_k$.

Next, some operators Y^P and Y^S are introduced that perform projection H on W^P and H on W^S in n-dimensional space of R^n for the norms of $\left\|P_q(t,z)\right\|$ and $\left\|S_{2_q}(t,z)\right\|$. Then, the expressions of the dynamics (11), (12) can be represented as

$$-Y^P \frac{m\,\partial S_2}{\partial t} \geq Y^P A_1'\left(P_q, v, z\right) - Y^P A_1'\left(P_q, S_2, z\right) + Y^P f_1(t), \tag{17}$$

$$Y^S \frac{m\,\partial S_2}{\partial t} = Y^S A_1''\left(P_q, z\right) + Y^S f_2(t) \tag{18}$$

and here (17) and (18) are performed in the spaces of W^P and W^S almost for all $t \in (0,t_k)$. The above arguments imply that the operators $Y^P A_1'\left(P_q, v, z\right), Y^P A_1''\left(P_q, z\right)$ belong to the space boundary of $L^2\left(0,t_k,W^P\right)$, and the operator of $Y^P A_1'\left(P_q, S_2, z\right)$ — to the space boundary $L^2\left(0,t_k,W^S\right)$.

Finally, it follows that the required solution of $\{P(t,z),\ S_2(t,z)\}$ can be obtained from the approximate of $\{P_q(t,z),\ S_{2_q}(t,z)\}$, and thus the following conditions for convergence are taken into account:

— $P_q \to P$ in the space of $L^2\left(0,t_k,W^P\right)$ and $S_{2_q} \to S_2$ in the space of $L^2\left(0,t_k,W^S\right)$ — *weak*;

— $\dfrac{m\partial S_{2_q}}{\partial t} \to \dfrac{m\partial S_2}{\partial t}$ in the space of $L^2\left(0,t_k,W^S\right)$ — *weak*;

— $P_q \to P$ in the space of $L^\infty\left(0,t_k,W^P\right)$ and $S_{2_q} \to S_2$ in the space of $L^\infty\left(0,t_k,W^S\right)$ — *weak.*

Therefore, the implemented limiting transition is the proof that the set of $\{P(t,z),\ S_2(t,z)\}$ provided by the specified conditions of convergence, is a solution of the (1) — (5) system for the parameters of $\left(m(z),\ k_1(z)\right) \in \Lambda_d$.

The next phase of the qualitative analysis is the proof of the uniqueness of the problem solving of (1) — (5). Suppose that (1) — (5) has two solutions, defined by the set of $\left\{P^1(t,z),\ S_2^1(t,z)\right\}$ and $\left\{P^2(t,z),\ S_2^2(t,z)\right\}$. Then it may also be assumed that for each point of the domain Ω there are numbers

$$\eta^P = P^1(t,z) - P^2(t,z);\ \eta^S = S_2^1(t,z) - S_2^2(t,z).$$

Replacing in terms of the systems dynamics of (1) — (5) the functions of $P(t,z)$, $S_2(t,z)$ respectively by $P^1(t,z)$, $S_2^1(t,z)$ and $P^2(t,z)$, $S_2^2(t,z)$, it is possible to get two systems where the termwise subtraction will lead to the result

$$\left(\frac{m\partial\eta^S}{\partial t}\right)(v-\eta^S)-\left[A_1'(P^1,v,t,z)-A_1'(P^2,v,t,z)\right]+\left[A_1'(P^1,S_2,t,z)-A_1'(P^2,S_2,t,z)\right]\geq 0 \quad (19)$$

$$\left(\frac{m\partial\eta^S}{\partial t}\right)-\left[A_1''(P^1,t,z)-A_1''(P^2,t,z)\right]=0 \quad (20)$$

$$\eta^P(0)=0;\ \eta^S(0)=0 \quad (21)$$

The expressions in square brackets in (19) and (20), based on the definition of the operators $A_1'(\cdot)$ and $A_1''(\cdot)$, can be presented as

$$A_1'(P^1,v,t,z)-A_1'(P^2,v,t,z)=\int_\Omega\sum_{i=1}^n\left\{k_1\frac{\partial^2}{\partial z_i^2}\left[\beta(P^1)-\beta(P^2)\right]|v|\right\}dz\ ,$$

$$A_1'(P^1,S_2^1,t,z)-A_1'(P^2,S_2^2,t,z)=\int_\Omega\sum_{i=1}^n\left\{k_1\frac{\partial^2}{\partial z_i^2}\left[\beta(P^1)-\beta(P^2)\right]\left|S_2^1-S_2^2\right|\right\}dz\ ,$$

$$A_1''(P^1,t,z)-A_1''(P^2,t,z)=\sum_{i=1}^n\left\{k_1\frac{\partial^2}{\partial z_i^2}\left[\beta(P^1)-\beta(P^2)\right]\right\}dz\ .$$

Thus the system (19) — (21) can be presented as

$$\begin{gathered}\left|S_2^1-S_2^2\right|\left(\frac{m\partial\eta^S}{\partial t}\right)(v-\eta^S)-\int_\Omega\sum_{i=1}^n\left\{k_1\frac{\partial^2}{\partial z_i^2}\left[\beta(P^1)-\beta(P^2)\right]\left|P^1-P^2\right||v|\right\}dz+\\+\int_\Omega\sum_{i=1}^n\left\{k_1\frac{\partial^2}{\partial z_i^2}\left[\beta(P^1)-\beta(P^2)\right]\left|P^1-P^2\right|\left|S_2^1-S_2^2\right|\right\}dz\geq 0\end{gathered} \quad (22)$$

$$\left|S_2^1-S_2^2\right|\left(\frac{m\partial\eta^S}{\partial t}\right)-\sum_{i=1}^n\left\{k_1\frac{\partial^2}{\partial z_i^2}\left[\beta(P^1)-\beta(P^2)\right]\left|P^1-P^2\right|\right\}dz=0 \quad (23)$$

$$\eta^P(0)=0;\ \eta^S(0)=0 \quad (24)$$

It is obvious that implementing the conditions of the system (22) — (24) is possible only under the condition that $P^1(t,z)=P^2(t,z)$, $S_2^1(t,z)=S_2^2(t,z)$ which proves the uniqueness of the solution of initial problem (1) — (5).

Thus, there is the solution of the problem in $L^2(\Omega)\bigcap L^\infty(0,t_k,H)$ and it is unique.

Now the differentiability of the functional (10) in the form of proving the following theorem is under study.

Theorem 2. For $(m(z),k_1(z))\in\Lambda_d$ the functional (10) has a weak derivative Λ_d (i.e., the derivative in the sense of Gateaux) in the domain R^n.

Proof. The functional derivative $J_1[m(z),k_1(z)]$ is defined as

$$\begin{gathered}\delta J_1(m,k_1)=\int_\Omega\delta m(z)\left\{\left[\frac{\partial\left[P(t,z_j,m,k_1)-F_j^P(t)\right]}{\partial t}+\right.\right.\\\left.\left.+\frac{\partial\left[S_2(t,z_j,m,k_1)-F_j^S(t)\right]}{\partial t}\right]\cdot p^*dt\right\}dz+\int_\Omega\delta k_1(z)\left\{\sum_{j=1}^{K_1+K_2}\left[\frac{\partial\left[P(t,z_j,m,k_1)-F_j^P(t)\right]}{\partial z_j}+\right.\right.\\\left.\left.+\frac{\partial\left[S_2(t,z_j,m,k_1)-F_j^S(t)\right]}{\partial z_j}\right]\cdot\frac{\partial p^*}{\partial z_j}dt\right\}dz\end{gathered} \quad (25)$$

where $\{P(t,z), S_2(t,z)\}$ is the solution of the problem (1) — (5), and $p^*(t,z)$ is the solution of the adjoint system of the form

$$-m(z)\frac{\partial p^*}{\partial t}(v-S_2)-\left[A_1'(P,v,t,z)+A_1'(P,S_2,t,z)\right]\sum_{i=1}^{n}\left[k_1(z)\frac{\partial p^*}{\partial z_i}\right]\geq 0 \quad (26)$$

$$-m(z)\frac{\partial p^*}{\partial t}-A_1''(P,t,z)\sum_{i=1}^{n}\left[k_1(z)\frac{\partial p^*}{\partial z_i}\right]=0 \quad (27)$$

$$\frac{\partial p^*(t,z)}{\partial t}=0 \quad \Sigma=\partial\Omega\times(0,t_k) \quad (28)$$

$$p^*(t_k,z)=0 \text{ for } \Omega \quad (29)$$

The conjugate function satisfies the following conditions

$$p^*(t,z)\in L^2(0,t_k,H(\Omega))\bigcap L^\infty(0,t_k) \quad \frac{\partial p^*(t,z)}{\partial t}\in L^2(\Omega) \quad (30)$$

Assuming that $(m(z), k_1(z))\to P(t,z)$ and $(m(z), k_1(z))\to S_2(t,z)$ are continuous on Λ_d and, consequently, have weak derivatives (ie, derivatives in the sense Gato) on $L^2(\Omega)$, we can prove that the criterion $J_1[m(z), k_1(z)]$ (8) is also Gateaux-differentiable, and its derivative Λ_d is

$$\delta J_1=-\int_0^{t_k}\int_\Omega\left[e^P(t,z)\delta P(t,z)+e^S(t,z)\delta S_2(t,z)\right]dz\,dt,$$

where

$$e^P(t,z)=-2\sum_{j=1}^{K_1+K_2}\left\{\frac{1}{|z_j|}\int_\Omega\left[P(t,z_j,m,k_1)-F_j^P(t)\right]dz\right\};$$

$$e^S(t,z)=-2\sum_{j=1}^{K_1+K_2}\left\{\frac{1}{|z_j|}\int_\Omega\left[S_2(t,z_j,m,k_1)-F_j^S(t)\right]dz\right\},$$

$\delta P(t,z)$ and $\delta S_2(t,z)$ — respectively increment of functions $P(t,z)$ and $S_2(t,z)$.

For the functions $\delta P(t,z)\in L^2(\Omega)$ and $\delta S_2(t,z)\in L^2(\Omega)$, given the accepted symbols, the expressions of the systems dynamics can be written

$$\left(\frac{\delta m(z)\partial\delta S_2}{\partial t}\right)(\delta v-\delta S_2)-A_1'(\delta P,\delta v,t,z)+A_1'(\delta P,\delta S_2,t,z)=$$

$$=\left(\frac{\delta m(z)\partial\delta S_2}{\partial t}\right)(\delta v-\delta S_2)-\int_\Omega\sum_{i=1}^{n}\left[\delta k_1(z)\frac{\partial\delta P}{\partial z_j}|\delta v|\right]dz+$$

$$\int_\Omega\sum_{i=1}^{n}\left[\delta k_1(z)\frac{\partial\delta P}{\partial z_j}|\delta S_2|\right]dz\leq f(t,z) \quad (31$$

$$\left(\frac{\delta m(z)\partial\delta S_2}{\partial t}\right)-A_1''(\delta P,t,z)=\left(\frac{\delta m(z)\partial\delta S_2}{\partial t}\right)-\sum_{i=1}^{n}\delta k_1(z)\frac{\partial^2\delta P}{\partial z_j^2} \quad (32)$$

Next, the function $\rho(t,z)\in L^2(\Omega)$ which can be defined by the following system is introduced into consideration

$$\left.\begin{aligned}\left(\frac{m\,\partial\rho}{\partial t}\right)-A_1'(\rho,v,t,z)+A_1'(\rho,S_2,t,z)=\int_\Omega\left(e^P\rho+e^S\rho\right)dt;\\ \rho(t_k)=0,\end{aligned}\right\} \quad (33)$$

and this system has a unique solution (which follows from the proof of Theorem 1), satisfying the conditions

$$\left.\begin{aligned} \rho \in L^2(\Omega)\bigcap L^{\infty}(0,t_k,H), \\ \frac{\partial \rho}{\partial t} \in L^2(\Omega). \end{aligned}\right\} \qquad (34)$$

Using the expressions (31) — (34) the following can be obtained

$$\delta J_1 = \left(\delta m(z)\frac{\partial \delta S_2}{\partial t}\right)(\delta v - \delta S_2) - \int_{\Omega}\sum_{i=1}^{n}\left[\delta k_1(z)\frac{\partial \delta P}{\partial z_j}|\delta v|\right]dz + \int_{\Omega}\sum_{i=1}^{n}\left[\delta k_1(z)\frac{\partial \delta P}{\partial z_j}|\delta S_2|\right]dz \qquad (35)$$

By equating in turn $p^* = \delta S_2$ и $p^* = \delta P$, the inequality (26) can be obtained from (35) and the relation (30) from (33). In addition, (35) results from (25). Hence Theorem 2 is proved.

We can write the derivative $J_1(m,k_1)$ by the parameters in the form of $m(z)$ and $k_1(z)$

$$\delta J_1 = J_1(m,k_1)[\delta m(z), \delta k_1(z)] = \int_{\Omega}\left[\delta m(z)\frac{\partial J_1(m,k_1)}{\partial m(z)} + \delta k_1(z)\frac{\partial J_1(m,k_1)}{\partial k_1(z)}\right]dz,$$

Where

$$\frac{\partial J_1(m,k_1)}{\partial m(z)} = \int_0^{t_k}\frac{\partial S_2(t,z)}{\partial t}[v - S_2(t,z)]p^*(t,z)dt \qquad (36)$$

$$\frac{\partial J_1(m,k_1)}{\partial k_1(z)} = \int_0^{t_k}\left[A_1'(P,v,t,z) + A_1'(P,S_2,t,z)\right]\sum_{i=1}^{n}\frac{\partial P(t,z)}{\partial z_i}\frac{\partial p^*(t,z)}{\partial z_i}dt \qquad (37)$$

The relations (36), (37) are convenient for the numerical calculation of the gradient of the functional $J_1[m(z), k_1(z)]$ while writing the optimization problem in the form (10).

A qualitative analysis of the problem of identification for water-oil reservoirs by the porosity and permeability parameters, which are the parameter settings of the MM for filtering process of the anomalous fluid, showed that there is a solution for the formulated problem of identification in the optimization setting and it is unique. In addition, the quality criteria in the formulated optimization problem is differentiable with respect to identifiability of the porosity and permeability parameters, which means the possibility to achieve extremum in its decision. In other words, a quality criterion for the optimization problem can be minimized by MM settings, and the initial problem of identification for water-oil reservoirs is the correct solution.

Conclusion

The statement of the problem of parametric identification of MM of multicomponent anomalous diffusion processes in the form of an optimal control problem is carried out.

A qualitative study of the problem of parametric identification of MMs of multicomponent anomalous diffusion processes has been carried out, during which existence and uniqueness theorems for the optimization problem posed have been proved. The differentiability of the accepted quality criterion with respect to the MM settings is also proved.

An approach to solving the problem of parametric identification of MMs of multicomponent anomalous diffusion processes, presented in an optimization formulation, is proposed. The approach is based on the procedure of the gradient projection method. The possibility of applying the proposed approach to solving problems both in linear and non-linear formulations is substantiated.

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ДОСЛІДЖЕННЯ МОЖЛИВОСТІ РОЗВ'ЯЗУВАННЯ ЗАДАЧІ ІДЕНТИФІКАЦІЇ ВОДОНАФТОВИХ СУМІШЕЙ ПО ПАРАМЕТРАХ НАЛАШТУВАННЯ МАТЕМАТИЧНОЇ МОДЕЛІ

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Виконано постановку задачі параметричної ідентифікації для водо-нафтових пластів у випадку, коли одна з рідин, що фільтрується, має аномальний характер. При цьому задачу ідентифікації сформульовано як задачу оптимального управління, що зводиться до відшукання екстремуму критерію якості (функціонала). Одержано умови існування та єдиності розв'язку задачі ідентифікації за параметрами налаштування математичної моделі, а також диференційованості критерію якості, у зв'язку з чим доведено відповідні теореми. Розв'язок задач моделювання та ідентифікації аномальних процесів дифузії пов'язане з низкою важливих складностей як постановочного, так і обчислювального характеру. У цьому сенсі слід зазначити, зокрема: нелінійний характер досліджуваних процесів; складність геометрії просторової області моделювання та її границь; обмеженість вектора вимірювань простору станів процесу та числа точок прикладення управляючих впливів; високі розмірності результуючих кінцевовимірних аналогів математичної моделі (ММ). Не набули достатнього розвитку математичні методи опису аномальних дифузійних процесів при мультиплікативному представленні функцій стану, а також для випадку багатокомпонентних дифузних систем. Зазначені проблеми потребують як розвитку, так і розробки нових методів дослідження аномальних дифузійних процесів.Різко виражена спрямованість розвитку аномальних дифузійних процесів зумовлює адекватність їх математичної формалізації на основі апарату варіаційних нерівностей.

Ключові слова: математична модель, варіаційні нерівності, математичний метод, моделювання та ідентифікація.